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Journal of APPLIED MATHEMATICS AND MECHANICS
ELSEVIER
www.elsevier.com/locate/jappmathmech
Journal of Applied Mathematics and Mechanics 69 (2005) 618–627

DYNAMIC THERMOELECTROELASTIC FIELDS NEAR MOVING NON-SMOOTH INTERFACES OF MEDIA†

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(*Received* 24 *December* 2003)

The asymptotic behaviour of dynamic thermal, mechanical and electromagnetic fields near a singular line (a set of corner points) of a moving interface of thermoelectrically conducting media when they are subject to external dynamic thermoelectromechanical actions is established. © 2005 Elsevier Ltd. All rights reserved.

The elastic fields in bodies under condition of generalized plane deformation with a non-smooth moving interface of the boundary conditions were investigated in [1].

An analytical investigation of the thermoelastic state of electrically conducting non-uniform three-dimensional bodies with non-smooth interfaces presupposes the construction of functions defining the corresponding fields. The class of such functions is determined by their behaviour near discontinuities of the boundary surfaces. A knowledge of the asymptotic form of the functions enables us to construct solutions of problems on the thermoelastic state of three-dimensional electrically conducting bodies in the same way as for the three-dimensional fields described by harmonic functions in regions with non-smooth boundaries [2].

1. FORMULATION OF THE PROBLEM

Suppose $S = S_1 \cup S_2$ is the interface between two thermoelastic electrically conducting bodies. The intersection of the smooth surfaces S_1 and S_2 defines a smooth singular line $L = S_1 \cap S_2$, which is a set of corner points. We will introduce local curvilinear orthogonal coordinates ρ, θ, s by the relation [2]

$$r = r_0 + \rho(\cos\theta \mathbf{n}_2(s) + \sin\theta \mathbf{n}(s)) \tag{1.1}$$

where r_0 and r are the radius vectors of the points $M_0 \in L$ and M, the point M lies in the plane of the vectors $\mathbf{n}\mathbf{n}_2$ of the moving trihedron $\mathbf{n}\mathbf{n}_1\mathbf{n}_2$ at the point M_0, drawn on the singular line of the surface S_0, and is defined in this plane by the polar radius ρ and the angle θ, $\mathbf{n}_1$ is the tangential vector to the singular line, $\mathbf{n}_2$ is the tangential vector to the drawn surface and s is the length of the arc of the singular line measured from a certain point on it.

The elastic electrically conducting bodies considered to do not possess spontaneous polarization and magnetization and are subjected to the action of a mechanical load and a temperature field, and are also situated in a varying electromagnetic field. There are no external electric charges and currents on the contact surface, and the materials of the bodies have constant characteristics.

Taking into account the effect of the field of the electric potential on the deformation process, the thermal conductivity and the electrical conductivity, we will choose as the governing quantities the displacement vector $\mathbf{u}$, the temperature T and the electric potential Φ.

†*Prikl. Mat. Mekh.* Vol. 69, No. 4, pp. 684–694, 2005.

0021-8928/$—see front matter.
doi: 10.1016/j.jappmathmech.2005.07.011

The equations of equilibrium, heat conduction, conservation of electric charges and Maxwell's equations must be satisfied at all points of the composite body, including at points on the singular line [3]. We have

$$\lim\left(c_{1j}^2\operatorname{grad}\operatorname{div}\mathbf{u}_j - c_{2j}^2\operatorname{rot}\operatorname{rot}\mathbf{u}_j - a_{4j}\operatorname{grad}T_j - a_{5j}\operatorname{grad}\Phi_j - \frac{\partial^2\mathbf{u}_j}{\partial t^2}\right) = 0$$

$$\lim\left(\Delta T_j - b_{4j}\frac{\partial T_j}{\partial t} - b_{5j}\frac{\partial\operatorname{div}\mathbf{u}_j}{\partial t} + b_{6j}\frac{\partial\Phi_j}{\partial t}\right) = 0 \tag{1.2}$$

$$\lim\left(\Delta\Phi_j - d_{4j}\frac{\partial\Phi_j}{\partial t} + d_{5j}\frac{\partial T_j}{\partial t} - d_{6j}\frac{\partial\operatorname{div}\mathbf{u}_i}{\partial t} - d_{7j}\Phi_j + d_{8j}T_j - d_{9j}\operatorname{div}\mathbf{u}_j\right) = 0$$

$$\lim\left(\operatorname{rot}H_j - \varepsilon_j\frac{\partial E_j}{\partial t} - \lambda_j(E_j - \operatorname{grad}\Phi_j)\right) = 0$$

$$\lim\left(\operatorname{rot}E_j + \mu_{0j}\frac{\partial H_j}{\partial t}\right) = 0, \quad \lim\operatorname{div}H_j = 0$$

$$\lim(\varepsilon_{0j}\operatorname{div}E_j - \rho_{0j}C_{0j}(\Phi_j - \gamma_jT_j) - \beta_jK_j\operatorname{div}\mathbf{u}_j) = 0$$

$$c_{1j}^2 = \frac{\lambda_j + 2\mu_j}{\rho_{0j}}, \quad c_{2j}^2 = \frac{\mu_j}{\rho_{0j}}, \quad a_{4j} = \frac{\alpha_jK_j}{\rho_{0j}}, \quad a_{5j} = \frac{\beta_jK_j}{\rho_{0j}}, \quad b_{4j} = \frac{\rho_{0j}c_j}{\kappa_j}$$

$$b_{5j} = \frac{\alpha_jK_jT_{0j}}{\kappa_j}, \quad b_{6j} = \frac{\rho_{0j}\gamma_jC_{0j}T_{00}}{\kappa_j}, \quad d_{4j} = \frac{\rho_{0j}C_{0j}}{\lambda_j}, \quad d_{5j} = \frac{\rho_{0j}C_{0j}\gamma_j}{\lambda_j}$$

$$d_{6j} = \frac{\beta_jK_j}{\lambda_j}, \quad d_{7j} = \frac{\rho_{0j}C_{0j}}{\varepsilon_{0j}}, \quad d_{8j} = \frac{\rho_{0j}C_{0j}\gamma_j}{\varepsilon_{0j}}, \quad d_{9j} = \frac{\beta_jK_j}{\varepsilon_{0j}}$$

$$\lambda_j = \frac{\nu_jE_j}{(1+\nu_j)(1-2\nu_j)}, \quad K_j = \frac{3\lambda_j + 2\mu_j}{3}$$

$\mathbf{u}_j$ is the displacement vector, T_j is the temperature field, Φ_j is the electric potential, $\mathbf{H}_j$ and $\mathbf{F}_j$ are the electric and magnetic field vectors, c_j is the specific heat capacity, α_j is the temperature coefficient of volume expansion, ν_j is Poisson's ratio, μ_j and E_j are the shear modulus and modulus of elasticity, ρ_{0j} is the density of the material, T_{00} is a constant, β_j is the electrostriction coefficient of volume expansion, κ_j is the thermal conductivity, λ_j is the electrical conductivity, γ_j is the temperature coefficient of variation of the electrical potential, ε_{0j} and μ_{0j} are the dielectric constant and magnetic permeability, c_{1j} and c_{2j} are the velocities of the expansion and shear waves of the medium, C_{0j} is the specific electrical capacitance, and the values of the subscript $j = 0, 1$ denote the bodies which comprise the non-uniform body. The limits are taken as $M \to M_0 \in L$.

We will assume that, in the model considered, at points of the singular line of the interface of the media the conditions of thermoelectromechanical contact are satisfied in the form

$$\begin{gathered}\lim(u_{\rho0} - u_{\rho1}) = 0, \quad \lim(u_{\theta0} - u_{\theta1}) = 0, \quad \lim(u_{s0} - u_{s1}) = 0 \\ \lim(\sigma_{\theta0} - \sigma_{\theta1}) = 0, \quad \lim(\tau_{\rho\theta0} - \tau_{\rho\theta1}) = 0, \quad \lim(\tau_{s\theta0} - \tau_{s\theta1}) = 0\end{gathered} \tag{1.3}$$

$$\lim(T_0 - T_1) = 0, \quad \lim(q_{\theta0} - q_{\theta1}) = 0 \tag{1.4}$$

$$\lim(\Phi_0 - \Phi_1) = 0, \quad \lim(j_{\theta0} - j_{\theta1}) = 0 \tag{1.5}$$

$$\lim(E_{\theta0} - E_{\theta1}) = 0, \quad \lim(E_{s0} - E_{s1}) = 0 \tag{1.6}$$

$$\lim(H_{\theta0} - H_{\theta1}) = 0, \quad \lim(H_{s0} - H_{s1}) = 0 \tag{1.7}$$

where the limits are taken as $M \to M_0$ in the plane normal to the singular line, $q_{\theta j}$ and $j_{\theta j}$ are the components of the heat flux density vector and the conduction current density vector, respectively, $\sigma_{\theta j}$,

$\tau_{\rho\theta j}$, $\tau_{\theta sj}$ are the components of the stress tensor, and $E_{\theta j}$, H_{sj}, $H_{\theta j}$, H_{sj} are the components of the magnetic field vectors.

Suppose the translational displacement of a local part of the interface between the media is defined, as a rigid whole, by the vector $r_0(t) = \{x_0(t), y_0(t), z_0(t)\}$ in a fixed system of coordinates $Oxyz$.

Conditions in the form (1.3) enable us to investigate certain cases in a single way, for example, the interaction of thermoelectromechanical fields near the singular line of a propagating cavity filled with an ionized gas ($\sigma_{\theta 1} = \tau_{\rho\theta 1} = \tau_{\theta s1} = 0$) [4] and the interaction of thermoelectroelastic fields in the contact problem of a rigid and elastic body [5].

We will obtain, in the local neighbourhood of the singular line, the distribution of the components of the stress tensor, the temperature field, the heat flux density, the conduction current and the components of the electromagnetic field vectors, if the composition is subjected to external dynamic force, thermal and electromagnetic actions.

The initial conditions are not specified, since, as will be shown below, when Eqs (1.2) are satisfied a steady state will exist.

2. LOCAL THERMOELECTROELASTIC FIELDS

We will change in Eqs (1.2) to moving coordinates x_1, y_1, z_1, connected with the moving part of the interface by the formulae

$$x = x_1 + x_0(t), \quad y = y_1 + y_0(t), \quad z = z_1 + z_0(t)$$

The derivatives with respect to t in the new coordinates have the form

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = (\mathbf{v}_0, \nabla)(\mathbf{v}_0, \nabla)\mathbf{u}_1 - (\mathbf{v}_0, \nabla)\frac{\partial \mathbf{u}_1}{\partial t} - \frac{\partial((\mathbf{v}_0, \nabla)\mathbf{u}_1)}{\partial t} + \frac{\partial^2 \mathbf{u}_1}{\partial t^2} \tag{2.1}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{v}_0, \nabla)\mathbf{u}_1 + \frac{\partial \mathbf{u}_1}{\partial t} \tag{2.2}$$

where

$$\mathbf{v}_0 = \left\{\frac{dx_0}{dt}, \frac{dy_0}{dt}, \frac{dz_0}{dt}\right\}, \quad |\mathbf{v}_0| < c_{10}, \quad |\mathbf{v}_0| < c_{20}, \quad \mathbf{u}_1 = \mathbf{u}_1(x_1, y_1, z_1, t)$$

Here and henceforth, when solving system (1.2), we will omit the identifying subscript for simplicity.

The displacement of points of the elastic medium, the temperature field and the electric potential in the local region of the singular line, starting from the fact that they are equal to zero at points on the singular line, and also taking into account their form in the time-independent case [6–8], we will represent it in powers of the variable ρ of the local coordinates (1.1)

$$u_{x_1} = \rho^{m_1}A(\rho, \theta, s, t), \quad u_{y_1} = \rho^{m_2}B(\rho, \theta, s, t), \quad u_{z_1} = \rho^{m_5}C(\rho, \theta, s, t) \tag{2.3}$$

$$T = \rho^{m_6}T_3(\rho, \theta, s, t) + \rho^{m_7}T_4(\rho, \theta, s, t) \tag{2.4}$$

$$\Phi = \rho^{m_8}\Phi_3(\rho, \theta, s, t) + \rho^{m_9}\Phi_4(\rho, \theta, s, t) \tag{2.5}$$

where $m_q = m_q(s, t)(q = 1, 2, 5, 6, 7, 8, 9), A, B, C, T_3, T_4, \Phi_3, \Phi_4$ are bounded and continuous functions of their variables.

The electromagnetic field strengths near the singular line will also be sought in the form of a power series

$$\begin{gathered}\mathbf{E} = \{E_{21}, E_{22}, E_{23}\}, \quad \mathbf{H} = \{H_{21}, H_{22}, H_{23}\} \\ E_{2r} = \rho^{m_{1r}-1}E_1(\rho, \theta, s, t), \quad H_{2r} = \rho^{m_{1r+3}}H_1(\rho, \theta, s, t), \quad r = 1, 2, 3\end{gathered} \tag{2.6}$$

where $m_q = m_q(s, t) \in (0, 1)(q = 11, 12, \ldots, 16)$.

Representation (2.6) in the two-dimensional case of a body with a crack is identical with the well-known representation in [6].

We will write the equilibrium equations (1.2) in the new variables, taking into account relations (2.1) and (2.2), and change to local coordinates, taking representations (2.3)–(2.5) into account.

As a result, equating the expressions of like powers of ρ, we obtain a system of differential equations, which are compatible when $m_1 = m_2 = m$ and can be split into the following system

$$a_0A + a_1\frac{dA}{d\theta} + a_2\frac{d^2A}{d\theta^2} + b_0B + b_1\frac{dB}{d\theta} = 0$$
$$c_0A + c_3\frac{dA}{d\theta} + d_0B + d_1\frac{dB}{d\theta} + d_2\frac{d^2B}{d\theta^2} = 0 \qquad (2.7)$$

and the equation

$$g_2\frac{d^2C}{d\theta^2} + g_1\frac{dC}{d\theta} + g_0C = 0 \qquad (2.8)$$

Here

$$a_0 = c_{10}^2(m^2-1) - \upsilon_{01}^2 m(m-1) + \upsilon_{02}^2 m(m-1),\quad a_1 = (2m-1)\upsilon_{01}\upsilon_{02},\quad a_2 = c_{20}^2 - \upsilon_{02}^2$$

$$b_0 = \upsilon_{01}\upsilon_{02}(m-1),\quad b_1 = c_{10}^2(m-1) - c_{20}^2(m+1) + 2\upsilon_{02}^2$$

$$c_0 = (1-m-m^2)\upsilon_{01}\upsilon_{02},\quad c_3 = c_{10}^2(m+1) - c_{20}^2(m-1) - \upsilon_{02}^2(m+2)$$

$$d_0 = c_{20}^2(m^2-1) + \upsilon_{02}^2,\quad d_1 = \upsilon_{01}\upsilon_{02}(1-m) + c_{10}^2,\quad d_2 = c_{10}^2 - \upsilon_{02}^2$$

$$g_0 = c_{20}^2 m_2^2 - \upsilon_{01}^2 m_5(m_5-1),\quad g = \upsilon_{01}\upsilon_{02}(1-2m_5),\quad g_2 = c_{20}^2 - \upsilon_{02}^2$$

and υ_{01} and υ_{02} are the components of the velocity vector of the part of the interface in ρ, θ, s coordinates.

By satisfying the heat conduction equation (1.2) using representations (2.3)–(2.5), as before we obtain that $m_7 = m$ and

$$\frac{\partial^2 T_3}{\partial\theta^2} + m_6^2 T_3 = 0,\quad \frac{\partial^2 T_4}{\partial\theta^2} + m^2 T_4 = -b_5 f_6(\theta) \qquad (2.9)$$

where

$$f_6(\alpha) = \upsilon_{01}(m^2-1)A + \upsilon_{01}(m-1)\frac{\partial B}{\partial\theta} + \upsilon_{02}(m+1)\frac{\partial A}{\partial\theta} + \upsilon_{02}\frac{\partial^2 B}{\partial\theta^2}$$

Substitution (2.3)–(2.5) into the equations of conservation of electric charge (1.2), we obtain that $m_9 = m$ and

$$\frac{\partial^2\Phi_3}{\partial\theta^2} + m_8\Phi_3 = 0,\quad \frac{\partial^2\Phi_4}{\partial\theta^2} + m^2\Phi_4 = -d_6 f_6(\theta) \qquad (2.10)$$

Taking representations (2.3)–(2.5) into account it follows from Maxwell's equations (1.2) that

$$m_{11} = m_{12} = m_{13} = m_{14} = m_{15} = m_{16} = m_0$$

and when the displacement velocity of the interface of the media is non-zero we obtain

$$E_r = 0,\quad H_r = 0;\quad r = 1, 2, 3 \qquad (2.11)$$

The solution of system of equations (2.7) has the form

$$\begin{aligned} A(\theta, s, t) &= 2\mathrm{Re}(C_{01}\exp(\delta_1\theta) + C_{03}\exp(\delta_3\theta)) \\ B(\theta, s, t) &= 2\mathrm{Re}(n_1 C_{01}\exp(\delta_1\theta) + n_3 C_{03}\exp(\delta_3\theta)) \end{aligned} \tag{2.12}$$

where

$$n_q = b_{30}\delta_q^3 + b_{20}\delta_q^2 + b_{10}\delta_q + b_0$$

and $\delta_q = \alpha_q + i\beta_q$ are the complex non-conjugate roots of the characteristic equation

$$\delta^4 + d_{30}\delta^3 + d_{20}\delta^2 + d_{10}\delta + d_{00} = 0, \tag{2.13}$$

where $b_{30}, b_{20}, b_{10}, b_{00}, d_{30}, d_{20}, d_{10}, d_{00}$ are quantities defined by $\upsilon_{01}, \upsilon_{02}, c_{10}, c_{20}$.
The solution of Eq. (2.8) is

$$\begin{aligned} &C(\theta, s, t) = \exp(\alpha_5\theta)(C_{51}\cos\beta_5\theta + C_{52}\sin\beta_5\theta) \\ &\alpha_5 = \upsilon_{01}\upsilon_{02}\frac{m_5 - 1/2}{c_{20}^2 - \upsilon_{01}^2}, \quad \beta_5 = \frac{\sqrt{|D_0|}}{2(c_{20}^2 - \upsilon_{01}^2)} \\ &|D_0| = 4(c_{20}^2 - (\upsilon_{01}^2 + \upsilon_{02}^2))c_{20}^2 m_5^2 + \upsilon_{01}^2(4m_5c_{20}^2 - \upsilon_{01}^2) \end{aligned} \tag{2.14}$$

It follows from Eqs (2.9) that

$$\begin{aligned} T_3(\theta, s, t) &= C_{61}\cos m_6\theta + C_{62}\sin m_6\theta \\ T_4(\theta, s, t) &= C_{71}\cos m\theta + C_{72}\sin m\theta + 2\mathrm{Re}(h_{71}\exp(\delta_1\theta) + h_{73}\exp(\delta_3\theta)) \end{aligned} \tag{2.15}$$

where

$$h_{7k} = C_{01}\frac{g_{7k}}{m^2 + \delta_k^2}$$

$$g_{7k} = -b_5(\upsilon_{01}(m^2 - 1) + \upsilon_{01}(m-1)n_k\delta_k + \upsilon_{02}(m+1)\delta_k + \upsilon_{02}n_k\delta_k^2); \quad k = 1, 3$$

and C_{61}, C_{62}, C_{71} and C_{72} are constants.
The representations of Φ_3 and Φ_4 follow from Eqs (2.10)

$$\begin{aligned} \Phi_3(\theta, s, t) &= C_{81}\cos m_8\theta + C_{82}\sin m_8\theta \\ \Phi_4(\theta, s, t) &= C_{91}\cos m\theta + C_{92}\sin m\theta + 2\mathrm{Re}(h_{91}\exp(\delta_1\theta) + h_{93}\exp(\delta_3\theta)) \end{aligned} \tag{2.16}$$

where

$$h_{9k} = C_{0k}\frac{g_{9k}}{m^2 + \delta_k^2}$$

$$g_{9k} = -d_6(\upsilon_{01}(m^2 - 1) + \upsilon_{01}(m-1)n_k\delta_k + \upsilon_{02}(m+1)\delta_k + \upsilon_{02}n_k\delta_k^2); \quad k = 1, 3$$

By satisfying the contact conditions (1.3) using representations (2.3)–(2.5) with the values

$$\begin{aligned} &\theta = \theta_g(s) = \arccos((\mathrm{grad} f_g, \mathrm{grad} f_0)(|\mathrm{grad} f_g||\mathrm{grad} f_0|)^{-1}) \\ &(0 \le \theta_g(s) \le \pi/2), \quad g = 1, 2 \end{aligned} \tag{2.17}$$

(the equation $f_q(x, y, z) = 0$ defines the surface S_q $(q = 0, 1, 2)$) at the point $M_0 \in L$, we arrive at the following system of equations

$$A_1 = A_0,\quad B_1 = B_0,\quad g_{01}^{(0)}\frac{\partial B_0}{\partial\theta} + g_{02}^{(0)}A_0 = g_{11}^{(0)}\frac{\partial B_1}{\partial\theta} + g_{12}^{(0)}A_1$$
$$h_{01}^{(0)}B_0 + h_{02}^{(0)}\frac{\partial A_0}{\partial\theta} = h_{11}^{(0)}B_1 + h_{12}^{(0)}\frac{\partial A_1}{\partial\theta} \tag{2.18}$$

$$C_1 = C_0,\quad \gamma\frac{\partial C_0}{\partial\theta} = \frac{\partial C_1}{\partial\theta} \tag{2.19}$$

Here

$$\gamma = \frac{\mu_0}{\mu_1},\quad g_{01}^{(0)} = \frac{1}{2}\gamma\mu_{00},\quad g_{02}^{(0)} = \gamma\left(1 + \frac{(m+1)\nu_0}{1-2\nu_0}\right),\quad g_{11}^{(0)} = \frac{1}{2}\mu_{01},\quad g_{12}^{(0)} = 1 + \frac{(m+1)\nu_1}{1-2\nu_1}$$

$$h_{01}^{(0)} = \gamma(m-1),\quad h_{02}^{(0)} = \gamma,\quad h_{11}^{(0)} = m-1,\quad h_{12}^{(0)} = 1,\quad \mu_{0j} = 2\frac{1-\nu_j}{1-2\nu_j}$$

By satisfying the thermal-contact conditions (1.4) and taking representation (2.4) into account, we obtain the following system of equations

$$T_{q1} = T_{q0},\quad \gamma_6\frac{\partial T_{q0}}{\partial\theta} = \frac{\partial T_{q1}}{\partial\theta};\quad q = 3, 4;\quad \gamma_6 = \frac{\kappa_0}{\kappa_1} \tag{2.20}$$

From the conditions for electrical contact (1.5) and taking representation (2.5) into account, we have the following system of equations

$$\Phi_{q1} = \Phi_{q0},\quad \gamma_8\frac{\partial\Phi_{q0}}{\partial\theta} = \frac{\partial\Phi_{q1}}{\partial\theta},\quad q = 3, 4;\quad \gamma_8 = \frac{\lambda_0}{\lambda_1} \tag{2.21}$$

Substituting representations (2.15) into system (2.7), we obtain a system of linear homogeneous algebraic equations in the constants C_{01}, C_{03}, C_{11} and C_{13}. Equating its determinant to zero, we obtain a characteristic equation for determining the order of the singularity of the dynamic stresses

$$\det Q = 0 \tag{2.22}$$

$Q = (p_{rq})$, $r, q = 1, 2, \dots, 8$; the rth row of the matrix Q is as follows:

$$(p_{rq})_q = (s_{1r1}\exp(\delta_{11}\theta_p),\ \bar{s}_{1r1}\exp(\bar{\delta}_{11}\theta_p),\ s_{1r3}\exp(\delta_{13}\theta_p),\ \bar{s}_{1r3}\exp(\bar{\delta}_{13}\theta_p),$$

$$s_{0r1}\exp(\delta_{01}\theta_p),\ \bar{s}_{0r1}\exp(\bar{\delta}_{01}\theta_p),\ s_{0r3}\exp(\delta_{03}\theta_p),\ \bar{s}_{0r3}\exp(\bar{\delta}_{03}\theta_p));\quad r = 1, 2, 3, 4$$

$$(s_{j1k}, s_{j2k}, s_{j3k}, s_{j4k})^T = (n_{jk}, 1, g_{jk}, h_{jk})^T,\ g_{jk} = g_{j1}^{(0)}\delta_{jk} + n_{jk}g_{j2}^{(0)},\ h_{jk} = h_{j1}^{(0)} + n_{jk}\delta_{jk}h_{j2}^{(0)};$$
$k = 1, 3$

$j = 1$ when $q = 1, 2, 3, 4$; $j = 0$ when $q = 5, 6, 7, 8$
$s_{jrp} = s_{jrp}, p = 1$ when $r = 1, 2, 3, 4$; $s_{jrp} = s_{jr-4p}, p = 2$ when $r = 5, 6, 7, 8$

n_{jk} are the complex non-conjugate roots of Eq. (2.13), written for the jth components and the bar denotes, as usual, a complex-conjugate quantity.

Using the solution of this system with zero determinant and representations (2.3)–(2.5) we can find the distribution of the local dynamic stresses and displacements in each body, making up the non-uniform body, close to the singular line of the moving part of the interface in local coordinates (1.1).

$$u_\rho^{(0)} = \sum_{n=1}^{l}(\rho^{m_n}M_{01n}K_n) + o(\rho^{m^*}),\quad u_\theta^{(0)} = \sum_{n=1}^{l}(\rho^{m_n}M_{02n}K_n) + o(\rho^{m^*}),\quad m^* = \max m_n$$

$$
\begin{aligned}
\sigma_\rho^{(0)} &= 2\mu_0 \sum_{n=1}^{l} \left(\rho^{m_n - 1}\left(((m_n + \nu_0(1 - m_n))(1 - 2\nu_0)^{-1} M_{01n} + \right.\right. \\
&\left.\left. + \nu_0(1 - 2\nu_0)^{-1}\frac{\partial M_{02n}}{\partial\theta}\right)K_n\right) + O(1) \\
\sigma_\theta^{(0)} &= 2\mu_0 \sum_{n=1}^{l} \left(\rho^{m_n - 1}\left((1 - 3\nu_0 + \nu_0 m_n)(1 - 2\nu_0)^{-1} M_{01n} + \right.\right. \\
&\left.\left. + (1 - \nu_0)(1 - 2\nu_0)^{-1}\frac{\partial M_{02n}}{\partial\theta}\right)K_n\right) + O(1) \\
\tau_{\rho\theta}^{(0)} &= 2\mu_0 \sum_{n=1}^{l} \left(\rho^{m_n - 1}\left((m_n - 1)M_{02n} + \frac{\partial M_{01n}}{\partial\theta}\right)K_n\right) + O(1)
\end{aligned}
\tag{2.23}
$$

where

$$
M_{01n} = 2\mathrm{Re}(n_{01}P_{0n}\exp(\delta_{01}\theta) + n_{03}S_{0n}\exp(\delta_{03}\theta))
$$

$$
M_{02n} = 2\mathrm{Re}(P_{0n}\exp(\delta_{01}\theta) + S_{0n}\exp(\delta_{03}\theta))
$$

l is the number of roots $m_n \in (0, 1)$ of the singular characteristic equation (2.22), P_{jn} and S_{jn} ($j = 0, 1$) are quantities determined by the elastic characteristics of the materials of the components, and K_n are the stress intensity factors. The distribution of the stresses and strains of the other component of the composite body has a form similar to distribution (2.23), taking its constants of elasticity into account.

By satisfying system (2.19), using (2.14), as previously we obtain the characteristic equation

$$
\left|\sin\left(\frac{1}{2}i\omega^*(\delta_{51} - \delta_{50})\right)\right| = q_5\left|\sin\left(\frac{1}{2}i\omega^*(\delta_{51} - \overline{\delta_{50}})\right)\right| \tag{2.24}
$$

where

$$
\omega^* = 2\pi - \omega, \quad \omega = \theta_1 + \theta_2, \quad q_5 = |\gamma\delta_{50} - \delta_{51}||\gamma\delta_{50} - \overline{\delta_{51}}|^{-1}
$$

and δ_{50} and δ_{51} are quantities in representation (2.14), written for both components of the composite body. The distribution of the corresponding displacements and stresses is one of the components of the composite body has the form

$$
\begin{aligned}
u_s^{(0)} &= \sum_{t=1}^{l_5} (\rho^{m_{5t}} M_{3t}K_{3t}) + o(\rho^{m_{50}}), \quad \tau_{\rho s}^{(0)} = \mu_0 \sum_{t=1}^{l_5} (m_{5t}\rho^{m_{5t} - 1} M_{3t}K_{3t}) + O(1) \\
\tau_{\theta s}^{(0)} &= \mu_0 \sum_{t=1}^{l_5} \left(K_{3t}\rho^{m_{5t} - 1}\frac{\partial M_{3t}}{\partial\theta}\right) + O(1), \quad M_{3t} = 2\mathrm{Re}(P_{3t}\exp(\delta_{50}\theta))
\end{aligned}
\tag{2.25}
$$

where l_5 is the number of roots $m_{5t} \in (0, 1)$ of the characteristic equation (2.24), $m_{50} = \max m_{5t}$. The distribution of the displacements and stresses in the other component of the composite body has a similar form, taking its constants of elasticity into account. Distribution (2.23) and characteristic equation (2.22) correspond to the plane dynamic problem, while distribution (2.25) and Eq. (2.24) correspond to the dynamic longitudinal shear problem.

Similarly, by satisfying systems (2.20) using representations (2.15), we obtain that

$$
m_6 = \pi/(2\pi - \omega) < 1 \tag{2.26}
$$

The temperature field in local curvilinear coordinates (1.1), based on the solutions obtained and representation (2.14) of one of the components of the composite body, has the form

$$T_0 = \rho^{m_6}(\gamma_6-1)^{-1}(k_{61}\cos m_6(\theta-\theta_1)+k_{62}\sin m_6(\theta-\theta_1))+$$
$$+\sum_{n=1}^{l}(\rho^{m_n}(G_{061n}\cos m_n\theta+G_{062n}\sin m_n\theta+F_{06n}(\theta)))+o(\rho^{m_6}) \tag{2.27}$$

where

$$F_{06n}(\theta) = 2\mathrm{Re}(C_{01}\exp(\delta_{01}\theta)(m_n^2+\delta_{01}^2)^{-1}+C_{02}\exp(\delta_{03}\theta)(m_n^2+\delta_{03}^2)^{-1}$$

G_{061n} and G_{062n} are determined when solving system (2.26), and k_{61} and k_{62} are the heat flux density intensity factors.

The temperature field distribution in the other component of the composition has a similar form, taking its thermal characteristics into account.

From systems (2.21), taking expressions (2.16) into account, we obtain the quantity which defines the order of the singularity of the conduction current density

$$m_8 = \pi/(2\pi-\omega)<1 \tag{2.28}$$

and, from the solution of the system of algebraic equations, we obtain the corresponding distribution of the electric potential in the matrix

$$\Phi_0 = \rho^{m_8}(\gamma_8-1)^{-1}(k_{81}\cos m_8(\theta-\theta_1)+k_{81}\sin m_8(\theta-\theta_1))+$$
$$+\sum_{n=1}^{l}(\rho^{m_n}(G_{081n}\cos m_n\theta+G_{081n}\sin m_n\theta+F_{06n}(\theta)))+o(\rho^{m_8}) \tag{2.29}$$

The constants G_{081n} and G_{082n} are found when solving system (2.28), and k_{81} and k_{82} are the conduction current density intensity factors.

The distribution in the other component of the non-uniform body also has a similar form, taking its electrical characteristics into account.

Hence, both the local stresses and the components of the heat flux density and the components of the conduction current density in the neighbourhood of the singular line have a power-form of singularity.

To obtain the case of a cavity filled, for example, with an ionized gas, propagating from the singular line, it is necessary to let μ_1 tend to zero in the singular characteristic equations (2.22) and (2.24), and in the distributions of the components of the local stress tensor (2.23) and (2.25). As a result we obtain

$$\left|\sin\left(\frac{1}{2}i\omega^*(\delta_{01}-\delta_{03})\right)\right| = q\left|\sin\left(\frac{1}{2}i\omega^*(\delta_{01}-\bar{\delta}_{03})\right)\right|,\quad \mathrm{Im}\delta_{50} = \pi/(2\pi-\omega) \tag{2.30}$$

where

$$q = |h_{03}g_{01}-h_{01}g_{03}||\bar{h}_{01}g_{03}-h_{01}\bar{g}_{03}|^{-1}$$

and the corresponding distribution of the components of the stress tensor.

Taking the limit in relations (2.22)–(2.24) as $\mu_1\to\infty$, which corresponds to the case of the first type of boundary conditions in the contact problem for absolutely rigid and elastic thermoelectrically conducting bodies [5], we obtain singular characteristic equations, which are identical with Eqs (2.30), and where it is necessary to put $q = |n_{01}-n_{03}|\,|n_{01}-\bar{n}_{03}|^{-1}$, and also the corresponding local stress distribution.

The orders of the singularity of the temperature field (2.26) and of the electric potential (2.28) are determined solely by the geometry of the interface of the media.

Letting the quantity κ_1 tend to zero (infinity), we obtain the distribution of the temperature field for the thermally insulated (absolutely heat conducting) part of the interface of the media. For an electrical conductivity $\lambda_1=0$ or $\lambda_1\to\infty$, we obtain from distribution (2.29) the distribution of the electric potential in the neighbourhood of the dielectric or absolutely conducting part of the interface.

3. A FIXED INTERFACE OF THE MEDIA

Assuming $\upsilon_{01} = 0$ and $\upsilon_{02} = 0$ in relations (2.22) and (2.24), we obtain the well-known characteristic equations, which determine the order of the singularity of the dynamic stresses near the fixed interface of the media [9]. In this case terms containing singularities of the stresses $1 - m_n$ $(n = 1, 2, 3, 4)$ in the distribution of the temperature field (2.27) and of the electric potential (2.29) disappear, and they are converted into distributions for the time-independent case.

The components of the electric field vector, as follows from relations (2.6), have power singularities and the following distribution in the matrix

$$E_{0\rho} = \rho^{m_0 - 1} k_{013} \sin m_0(\theta - \theta_1) + O(1)$$

$$E_{0\theta} = \rho^{m_0 - 1} k_{013} \cos m_0(\theta - \theta_1) + O(1)$$

$$E_{0s} = O(1), \quad m_0 = \pi/(2\pi - \omega)$$

The components of the magnetic field have a similar form.

Hence, we also obtain from boundary condition (1.5) the relation between the conduction current density intensity factor and the electric field strength intensity factor

$$k_{82} = -(1 - \gamma_8)^2 k_{013} (2 m_0 \gamma_8)^{-1} \tag{3.1}$$

In the case of a fixed interface between the media with a singular line, the distribution of the local stresses and strains, the temperature field and the heat flux, the electric potential and the conduction current density turn out to be the same both for dynamic loads and for static loads and for time-independent thermal and electromagnetic influences. This result agrees with the well-known results in the literature [10, 11] in the case of cracks ($\mu_1 = 0$, $\omega(s) = 2\pi$) and strong dynamic influences.

In the case of a cavity, the nature of the singularity of the components of the stress tensor is identical with that obtained earlier in [1], while the singularity of the electric potential agrees with well-known data [12]. Passing to the case of a two-dimensional plate with a crack, we obtain a singularity of order 1/2 for the conduction current density, which agrees with the well-known result in [3].

The singularity components of the heat flux density and the conduction current density is the sum of two terms, one of them, of the order of $1 - m_0$, is due to the thermal and electrical properties of the composition, while the second, which is of the order of $1 - m_n$ $(n = 1, 2, 3, 4)$, is due to the elastic properties of the composite and is determined by the stress intensity factors. Thus the thermal, electrical and mechanical fields are connected with one another near the singularities of the surface of the moving part of the interface between the media, i.e. local deformations near the singular line lead to perturbations of the thermal and electric fields. In the case of a fixed interface, this effect disappears, but a singularity of the components of the electric field vectors of the order $1 - m_0$ appears, and the conduction current density experiences the influence of the electromagnetic fields according to relation (2.31).

The realization of relations (1.2) leads to systems of differential equations (2.7)–(2.10), not containing derivatives with respect to the variable t, i.e. the problem is solvable for the time-independent mode.

The value of the opening angle of the interface at points of the singular line affects the order of the singularity of the heat flux, and electric potential and the electromagnetic field strength. The elastic properties of the composite and the velocity of displacement of the part of the interface, as follows from the singular characteristic equations (2.22) and (2.24), determine the singularity of the stresses and, via these, affect the singularity of the heat flux density and the conduction current density.

We will illustrate the effect of displacement of part of the interface on the local stress state using the example of the value of $1 - m_5$, defining the singularity of the stresses $\tau_{\rho s}$, $\tau_{\theta s}$ near the singular line of the surface of a propagating cavity filled with an ionized gas. From Eq. (2.30) we have

$$m_5 = \frac{1}{2(1 - (\eta_1^2 + \eta_2^2))}\left(-\eta_1^2 + (\eta_1^4 + (1 - (\eta_1^2 + \eta_2^2))^{1/2}(\eta_1^2\eta_2^2 + \frac{4\pi^2}{\omega^{*2}}(1 - \eta_1^2)^2))^{1/2}\right), \quad \omega^* = 2\pi - \omega \tag{3.2}$$

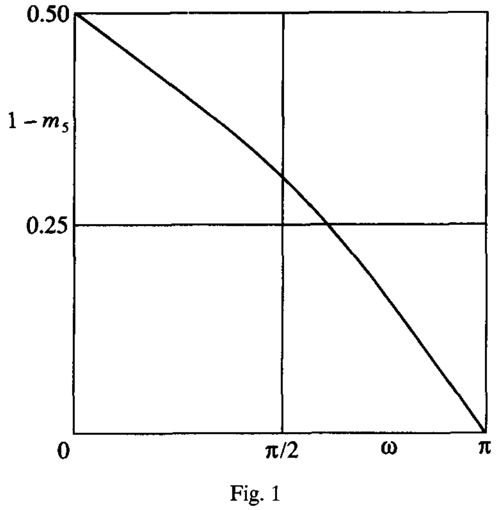

Fig. 1

where $h_1 = \upsilon_{01}c_{20}^{-1}$, $h_2 = \upsilon_{02}c_{20}^{-1}$ are relative values of the components of the propagating cavity velocity vector.

Numerical analysis of formula (3.2) shows that the values of η_1 and η_2, belonging to the interval (0.1, 1) have a considerable effect on the order of the singularity of the local stresses.

The change in the order of the singularity of the stresses $1 - m_5$ as a function of the opening angle ω of the surface of the propagating cavity at points of the singular line is shown in Fig. 1 for $\eta_1^2 = \eta_2^2 = 0.02$.

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Translated by R.C.G.